

THIS REPORT HAS BEEN DELIMITED
AND CLEARED FOR PUBLIC RELEASE
UNDER DOD DIRECTIVE 5200.20 AND
NO RESTRICTIONS ARE IMPOSED UPON
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.

ces Technical Information Agency

supply, you are requested to return this copy WHEN IT HAS SERVED
t it may be made available to other requesters. Your cooperation

7785

OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA
R THAN IN CONNECTION WITH A DEFINITELY RELATED
RATION, THE U. S. GOVERNMENT THEREBY INCURS
GATION WHATSOEVER; AND THE FACT THAT THE
TED, FURNISHED, OR IN ANY WAY SUPPLIED THE
OR OTHER DATA IS NOT TO BE REGARDED BY
SE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER
TION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE,
PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by
CUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED

OFFICE OF NAVAL RESEARCH

Contract N7onr-35801

T. O. I.

NR-041-032

THE EFFECT OF ACCELERATION TIME ON PLASTIC DEFORMATION
OF BEAMS UNDER TRANSVERSE IMPACT LOADING

by

D. S. Green

DIVISION OF APPLIED MATHEMATICS

BROWN UNIVERSITY

PROVIDENCE, R. I.

June, 1954

All-112

AD NO 37785
ASTIA FILE COPY

The Effect of Acceleration Time on Plastic
Deformation of Beams under Transverse Impact Loading¹

by

D. S. Green²

ABSTRACT

This paper considers a special case of point impact loading on a beam which is free to move in the direction of striking. The impact is such as to impart to the mid-point of the beam constant acceleration for a certain period, and then constant velocity for all subsequent time. If this acceleration is sufficiently great, the beam will undergo permanent deformation. The analysis shows how the magnitude of the deformation depends on the final velocity and the duration of the acceleration, assuming that the beam behaves in a perfectly plastic-rigid manner. This is the simplest problem introducing the finite time of acceleration that can be solved, and, although it is highly idealized, its solution will be valuable as a guide in the planning of experimental programs.

1. The results in this paper were obtained in the course of research conducted under Contract N7onr-35801 between Brown University and the Office of Naval Research.
2. Research Assistant, Division of Applied Mathematics, Brown University.

List of symbols

- m = mass/unit length of beam
 2ℓ = length of beam
 M_0 = fully plastic moment of beam
 x = distance along beam from mid-point
 t = time
 τ = duration of acceleration pulse
 t_s = time at which outer hinges disappear
 T = total time required to complete deformation
 v_0 = velocity of mid-point of beam
 v = velocity of point distant x from mid-point of beam
 V = final velocity of beam
 R = striking force applied at mid-point of beam
 M = bending moment at x
 λ = distance of outer hinge from mid-point of beam
 ξ = λ/ℓ
 ξ_0 = initial value of ξ
 K = $(1 - 2\xi)^4$
 θ = angle of rotation of inner segment
 Θ = final magnitude of θ ; when $t = T$, $\theta = \Theta$
 φ = angle of rotation of outer segment
 Φ = final magnitude of φ .

INTRODUCTION

The assumption of plastic-rigid behavior has been adopted several times in previous analyses of the plastic deformation of beams subjected to various types of impact loading [1 - 4]³. It is now desirable to verify its practical applicability by experiment. This can only be done if the theoretical solutions relate to experimental conditions. In impact tests it is easier, for example, to measure velocities than forces. With this in mind, Symonds and Leth [1] solved the problem in which the mid-point of a free beam is suddenly caused to move with a given velocity; they neglected, however, the effect of finite time of impact. Our present analysis, of which theirs is the special case of zero acceleration time, introduces this factor and therefore approaches more closely to real conditions. It is our intention here to provide information on the effects of acceleration and pulse time which will be useful in the arrangement and interpretation of tests.

The problem solved in this paper will now be specified. A uniform beam AB of length 2ℓ is at rest and without lateral restraint, when its mid-point O is struck laterally in such a way as to give to O constant acceleration for time τ , and constant velocity V thereafter (Fig. 1). It is required to find the resulting angle of deformation at the mid-point of the beam (θ in Fig. 2), assuming that the relation between bending moment and curvature is that shown in Fig. 3. Perfectly plastic-rigid

3. Numbers in square brackets refer to the Bibliography at the end of the paper.

mechanical behavior means that there are no elastic effects and the beam bends only where it is subjected to the fully plastic moment M_0 , at which sections it can assume unlimited curvature while M_0 is maintained. Thus elastic vibrations are precluded and the solution is conspicuously simplified. An analysis based on this assumption is likely to produce valid results for ductile materials only when the total energy absorbed in plastic deformation exceeds by far the total elastic energy the beam could store in bending [1,2].

There are three possible types of configuration that could arise from the motion imparted to the beam. Which of these obtains depends as follows on the magnitude of the non-dimensional quantity $m\ell^2 V/M_0 \tau$:

- (a) $m\ell^2 V/M_0 \tau \leq 2$, where m represents the mass per unit length of the beam. The limit moment M_0 is never achieved and hence there is no deformation.
- (b) $2 < m\ell^2 V/M_0 \tau \leq 41.7$ A plastic hinge appears at the mid-point and all the deformation is concentrated there.
- (c) $m\ell^2 V/M_0 \tau > 41.7$ Three plastic hinges occur, one at the mid-point and two symmetrically placed outer ones. The initial positions of the outer hinges depend upon V/τ , but these hinges move outwards until in the course of time they reach points a quarter of the length of the beam from the ends, and then the bending moment is no longer sufficient to sustain them and they disappear (Fig. 4). These outer hinges leave behind them smoothly-deformed regions.

Conditions are always symmetrical about the mid-point of the beam. After stating briefly the assumptions made to facilitate our analysis, we shall discuss cases (b) and (c) in detail. Case (a) is trivial.

Apart from the neglect of elastic effects the principal assumptions made are that the limit moment is unaffected by strain-rate [5], strain-hardening and shear stresses [6, 7]. Experiments are needed to tell us how important these phenomena are, and within what range our results are useful. It is perhaps fortunate that although strain-hardening and high strain-rate tend to increase the limit moment, shear stresses influence it in the contrary sense. A further assumption made throughout the work is that the changes of shape of the beam are so small as to have a negligible effect on the equations of motion.

ANALYSIS

1. Case of One Plastic Hinge ($2 < m\ell^2 V/M_0 \tau \leq 41.7$)

If V/τ is greater than $2M_0/m\ell^2$ the limit moment is always reached at the mid-point of the beam and a plastic hinge occurs there. This can easily be shown by considering the bending moment distribution caused by inertia forces in a rigid rod.

The equations for the rotation of either half of the beam (Fig. 5) during the pulse and for the subsequent time are respectively

$$\ddot{\theta} = \frac{3}{2} \frac{V}{\ell \tau} - \frac{3M_0}{m\ell^3} \quad (1)$$

and

$$\ddot{\theta} = - \frac{3M_0}{m\ell^3} \quad (2)$$

The plastic hinge disappears when $\dot{\Theta} = 0$. Application of this condition to (1) and (2) shows that the final angle of deformation developed by the beam is

$$\Theta = \frac{3}{4} \left(\frac{1}{2} - \frac{M_0 \tau}{m \ell^2 V} \right) \frac{m \ell V^2}{M_0} . \quad (3)$$

The value of $m \ell^2 V / M_0 \tau$ at which three plastic hinges first appear can be elicited from the bending moment distribution. Let M be the bending moment at a point distant x from 0. The equation for the translatory motion of the beam during the acceleration pulse is

$$\frac{R}{2} = m \ell \frac{V}{\tau} - \frac{m \ell^2}{2} \ddot{\Theta} \quad (4)$$

whence, using (1),

$$\frac{R}{2} = \frac{1}{4} m \ell \frac{V}{\tau} + \frac{3}{2} \frac{M_0}{\ell} \quad (5)$$

where R is the total force applied to the mid-point. The moment at 0 is the limit moment M_0 , therefore, reckoning sagging bending moment positive,

$$M = Rx/2 - M_0 - m \int_0^x (x - x') (V/\tau - x' \ddot{\Theta}) dx' . \quad (6)$$

By substituting for R and $\ddot{\Theta}$ and then finding the value of x at which M is a maximum, we find that the outer hinge first appears when $m \ell^2 V / M_0 \tau = 41.7$ and then $x/\ell = 0.404$.

In Table I are shown some values of the final angle Θ for corresponding values of $m \ell^2 V / M_0 \tau$ lying between 2 and 41.7. These have been plotted in Fig. 9.

TABLE I

$\frac{m\ell^2 V}{M_o \tau}$	2	3	4	5	10	20	30	41.7
$\frac{M_o \Theta}{m\ell V^2}$	0	0.125	0.188	0.225	0.300	0.338	0.351	0.357

2. Case of Three Plastic Hinges ($m\ell^2 V/M_o \tau > 41.7$)

If V/τ is greater than $41.7 M_o/m\ell^2$ three plastic hinges form simultaneously at the commencement of the motion. The initial positions of the symmetrically situated outer ones depend on the magnitude of V/τ . We shall now seek equations of motion for the beam by first taking any function $\dot{v}_o = \dot{v}_o(t)$ for the acceleration of the mid-point. These general equations can then be suitably adapted to represent each phase of the motion in turn (e.g. by putting $\dot{v}_o = V/\tau = \text{constant}$ for $0 < t \leq \tau$).

Conditions being symmetrical about the point O, we need consider only half of the beam, OA (Fig. 6). H represents the outer hinge in this half. The general equations of motion for OH and HA respectively are

$$2M_o = \frac{m\lambda^2}{2} \dot{v}_o - \frac{m\lambda^3}{3} \ddot{\theta} \quad (7)$$

$$M_o = \frac{m(\ell - \lambda)^3}{12} \ddot{\phi} \quad (8)$$

where $\lambda = \lambda(t)$ = distance of the outer hinge from the mid-point O;

θ = angle of rotation of the inner segment OH;

φ = angle of rotation of the outer segment HA.

The element of mass $m d\lambda$, which is transferred from the outer to the inner segment as the yield hinge traverses a distance $d\lambda$ does not influence the equations because its momentum is not affected by the transfer. Equation (8) follows from there being zero shear force at H, and so from the acceleration of the centre of mass of HA being zero. Further equations represent the velocity and acceleration distribution along the rod: for $x > \lambda$

$$v = v_0 - \int_0^{\lambda} \dot{\theta} dx - \int_{\lambda}^x \dot{\varphi} dx, \quad (9)$$

and therefore, by the rule for differentiation under integral signs,

$$\dot{v} = \dot{v}_0 - \int_0^{\lambda} \ddot{\theta} dx - \dot{\lambda} \dot{\theta} - \int_{\lambda}^x \ddot{\varphi} dx + \dot{\lambda} \dot{\varphi}. \quad (10)$$

But the acceleration of the mid-point of HA, $x = \frac{1}{2}(\lambda + \lambda)$ is zero, whence

$$\dot{v}_0 - \lambda \ddot{\theta} - \frac{1}{2}(\lambda - \lambda) \ddot{\varphi} + \dot{\lambda}(\dot{\varphi} - \dot{\theta}) = 0. \quad (11)$$

Elimination of $\ddot{\theta}$ and $\ddot{\varphi}$ from (11) gives

$$-\frac{\dot{v}_0}{2} + \frac{6M_0}{m\lambda^2} - \frac{6M_0}{m(\lambda - \lambda)^2} = -\dot{\lambda}(\dot{\varphi} - \dot{\theta}). \quad (12)$$

Having derived the general equations (7) to (12), let us investigate the motion and deformation of the beam in the following three phases:

- I during the acceleration pulse ($0 < t \leq \tau$);
- II the time between the end of the pulse and the elimination of the outer plastic hinges $H(\tau < t \leq t_s)$;
- III the time after this and until no more bending occurs at the mid-point $O(t_s < t \leq T)$.

It will be evident from our subsequent analysis that the outer hinges cease to be sustained before rotation ceases at the central one.

Phase I, $0 < t \leq \tau$

In this phase $\dot{v}_0 = V/\tau = \text{constant}$. It can be shown that in this circumstance (12) demands that $\dot{\lambda} = 0$ and so $\lambda = \lambda_0 = \text{constant}$ for $t \leq \tau$. Thus from (12), and putting $\lambda_0/\ell = \xi_0$,

$$\frac{12M_0\tau}{m\ell^2V} = \frac{\xi_0^2(1 - \xi_0)^2}{(1 - 2\xi_0)} \quad (13)$$

We have, therefore, that throughout the pulse period the hinges do not move along the rod; the greater the value of V/τ the closer are they to the mid-point; their initial positions can never be farther from the mid-point than 0.404ℓ because V/τ must exceed $41.7M_0/m\ell^2$. Figure 7 shows V/τ as a function of ξ_0 ; the curve is asymptotic to the line $\xi_0 = 0$. Henceforth in our work we shall express deformation as a function of ξ_0 instead of V/τ because of the consequent simplification. Figure 7 is a means of rapid transformation into terms of V/τ if such is required.

Returning to (7) and (8), for OH we have

$$\dot{\theta} = \left(\frac{3}{2} \frac{v}{\lambda_0 \tau} - \frac{6M_0}{m\lambda_0^3} \right) \tau \quad (14a)$$

$$= \frac{2(1 - 2\xi_0) - \xi_0^2}{2\xi_0(1 - 2\xi_0)} \frac{v}{\ell} \quad (\text{using (13)}) \quad (14b)$$

$$\theta_\tau = \frac{1}{2} \dot{\theta}_\tau \tau \quad (15)$$

and similarly for HA

$$\dot{\phi} = \frac{12M_0 \tau}{m(\ell - \lambda_0)^3} = \frac{\xi_0^2}{(1 - 2\xi_0)(1 - \xi_0)} \frac{v}{\ell} \quad (16)$$

$$\phi_\tau = \frac{1}{2} \dot{\phi}_\tau \tau \quad (17)$$

Substitution of values for ξ_0 in these equations gives the results representing conditions at $t = \tau$ in the first two lines of Table II, page 16. It is our object in this paper to determine the resultant deformation Θ , so only the figures relating to it have been tabulated, and those for ϕ have been omitted.

Phase II, $\tau < t \leq t_s$

The deflections and velocities at $t = \tau$ are the initial conditions for the differential equations of motion for this phase. The acceleration condition now is that $\dot{v}_0 = 0$, and so (7), (8) and (11) become (18), (19) and (20):

$$\ddot{\theta} = - \frac{6M_0}{m\ell^3} \frac{1}{\xi^3} \quad (18)$$

$$\ddot{\phi} = \frac{12M_0}{m\ell^3} \frac{1}{(1 - \xi)^3} \quad (19)$$

$$- \lambda \ddot{\theta} - \frac{(\ell - \lambda)}{2} \ddot{\phi} + \dot{\lambda}(\dot{\phi} - \dot{\theta}) = 0 \quad (20)$$

Elimination of $\ddot{\Theta}$ and $\ddot{\Phi}$ from these equations gives

$$\dot{\Phi} - \dot{\Theta} = \frac{6}{\xi} \left[\frac{1}{(1 - \xi)^2} - \frac{1}{\xi^2} \right] \quad (21)$$

Differentiation of (21) with respect to t , and then a further substitution for $\ddot{\Theta}$ and $\ddot{\Phi}$ yields the following equation relating ξ to t :

$$\frac{\ddot{\xi}}{\xi^2} = \frac{(1 - \xi)^2}{\xi(2\xi - 1)} \quad (22)$$

the solution to which is

$$\dot{\xi} \xi / [(1 - 2\xi)^{1/4} \exp(\xi/2)] = A \quad (23)$$

where A is a constant of integration depending on the angular velocities of the segments at $t = \tau$.

$$A = \frac{72M_0 C}{m\ell^2 V} \quad (24)$$

where

$$C = C(\xi_0) = (1 - 2\xi_0)^{7/4} / [6(1 - \xi_0) \{3(1 - 2\xi_0)(1 - \xi_0) - 2\xi_0^3 - 6(1 - \xi_0)^3\} \exp(\xi_0/2)] \quad (25)$$

The relation $C = C(\xi_0)$ is shown in Fig. 8. This phase of the problem is virtually solved now that (23) represents the movement of the outer hinges. It merely remains to find the increase in Θ during the period $\tau < t \leq t_s$, and the angular velocities $\dot{\Phi}$ and $\dot{\Theta}$ at t_s by integrating (18) and (19).

From (18) and (23),

$$\int_{\tau}^{t_s} \ddot{\Theta} dt = - \int_{\tau}^{t_s} \frac{6M_0}{m\ell^3} \frac{dt}{\xi^3} = \frac{2}{3} \frac{V}{\ell C} \int_{K_0}^0 (1 - K^4)^{-2} K^2 \exp\left[-\frac{1}{4}(1 - K^4)\right] dK \quad (26)$$

where $K = (1 - 2\xi)^{1/4}$ and K_0 corresponds to ξ_0 . Then

$$\int_{\tau}^{t_s} \ddot{\theta} dt = \frac{2}{3} \frac{V}{\ell C} \sum_{n=0}^{\infty} \int_{K_0}^0 (n+1) K^{(4n+2)} \exp\left[-\frac{1}{4}(1-K^4)\right] dK \quad (27)$$

The fact that when $t = t_s$, $\xi = \frac{1}{2}$ and $K = 0$ follows from (21) by putting $\dot{\varphi} = \dot{\theta}$, which is the condition for the disappearance of the outer hinges. Further, for the increase in θ during this phase we have

$$\int_{\tau}^{t_s} \dot{\theta} dt = \int_{\tau}^{t_s} (\dot{\theta} - \dot{\theta}_{\tau}) dt + \int_{\tau}^{t_s} \dot{\theta}_{\tau} dt \quad (28)$$

which has to be evaluated by the same means as was (26). The computed results are in Table II. The subscript s refers to conditions at $t = t_s$.

Phase III, $t_s < t \leq T$

Only the hinge at the mid-point now remains, and the two halves of the beam rotate as rigid bent rods. The equation of motion for OA is therefore

$$\ddot{\theta} = -3M_0/ml^3 \quad (29)$$

So the angular acceleration is constant. The rotation persists until the angular velocity is reduced to zero. Then $t = T$ and θ reaches its final value Θ . The results for this phase are also in Table II.

Resultant Deformation for Whole Period $0 < t < T$

Collecting together our data, we have Θ , the final angle at the mid-point (when three hinges have been present), given in Table II. All these results are subject to the angles being small, otherwise the equations of motion would be invalidated by changes of shape. The working has been checked by graphical integration to ensure that $\dot{\phi}_s = \dot{\Theta}_s$, and also by drawing up an energy balance for the case of $\xi_0 = 0.3$. Only Θ has been described here. Calculations show that $\Theta - \Phi$ is much less than Θ . Figure 2 indicates qualitatively the sort of deformation that occurs. It should be observed that most of the angle Θ is built up during the period $t_s < t \leq T$.

3. Results

All the values of Θ in Tables I and II (i.e. for both types of deformation configuration) have been plotted in Fig. 9, which contains the most important numerical results of the analysis. In the graph the line $M_0\Theta/mlV^2 = 0.425$ has been obtained from the work of Symonds and Leth [1] in which they considered the limiting case of infinite acceleration. This result can also be derived by the method used in the present paper: (13) shows that for $V/\tau \rightarrow \infty$, $\xi_0 \rightarrow 0$, and then suitable modification of (23) gives the required answer. It is a satisfying check on the computation for Fig. 9, that the graph of $M_0\Theta/mlV^2$ is asymptotic to this limiting line. It is worthy of note that, provided three hinges occur, our curve lies at all places within 15% of the limiting case and within this degree of approximation it is independent of V/τ .

4. Illustrative Examples

(a) For a $\frac{1}{4}$ " square steel bar 6" long, with mid-section accelerated to 50 ft/sec in 1/20 millisecc

$$m\ell^2 V/M_0 \tau \doteq 42$$

so we have the three hinge type of deformation. Figure 9 gives

$$\Theta = 0.358 m\ell V^2/M_0.$$

Therefore

$$\Theta \doteq 0.15 \text{ rdns.}$$

(Note: the limiting case of Symonds and Leth would give $\Theta = 0.18 \text{ rdns.}$).

(b) An 8" x $4\frac{1}{8}$ " @ 23 lbs. American Standard I beam 32 ft. long, with mid-section accelerated to 50 ft/sec in 1 millisecc, also deforms with three plastic hinges because

$$m\ell^2 V/M_0 \tau = 210$$

Figure 9 gives

$$\Theta = 0.395 m\ell V^2/M_0$$

$$\doteq 0.22 \text{ rdns.}$$

(Note: the limiting case of Symonds and Leth would give $\Theta = 0.24 \text{ rdns.}$)

CONCLUSIONS

Figure 9 contains the major results of this work. The type of deformation that occurs in a beam subjected to the loading specified in this problem is governed by the non-dimensional quantity $m\ell^2 V/M_0 \tau$. For $m\ell^2 V/M_0 \tau \leq 2$ no plastic deformation is caused. If $2 < m\ell^2 V/M_0 \tau \leq 41.7$ one hinge occurs at the mid-point and the damage can be ascertained for any particular case by using Fig. 9. When, however, $m\ell^2 V/M_0 \tau > 41.7$ the quantity $M_0 \Theta / m\ell V^2$ is substantially constant and an approximation to within 10% is that

$$\Theta = 0.4 m\ell V^2 / M_0 \quad m\ell^2 V/M_0 \tau > 41.7 \quad (30)$$

Thus Θ may be considered to be independent of τ in this region and to depend only on the square of the velocity V .

Experiments are now required for comparison with the results of this paper. It will then be known how influential are the various effects neglected in the analysis, chief among which are elastic vibrations, shear forces, strain-hardening, and strain-rate.

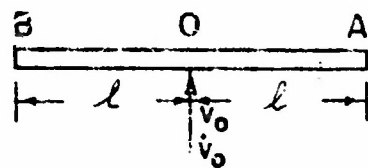
TABLE II

ξ_0	0.10	0.15	0.20	0.25	0.30	0.35	0.375	0.40	0.404
$\frac{M_0}{m\lambda^2} \theta_\tau$	0.00419	0.00635	0.00859	0.01099	0.01359	0.01634	0.01755	0.018	0.01804
$\lambda \frac{\dot{\theta}_\tau}{V}$	9.938	6.560	4.833	3.75	2.958	2.274	1.917	1.5	1.425
$\frac{M_0}{m\lambda^2} (\theta_s - \theta_\tau)$	0.0720	0.0633	0.0543	0.0445	0.0332	0.0199	0.0116	0.0017	0
$\lambda \frac{(\ddot{\theta}_s - \ddot{\theta}_\tau)}{V}$	-8.517*	-5.140	-3.412	-2.328	-1.536	-0.850	-0.493	-0.075	0
$\lambda \frac{\dot{\theta}_s}{V}$	1.418*	1.420*	1.421	1.422	1.422	1.423	1.424	1.425	1.425
$\frac{M_0}{m\lambda^2} (\theta - \dot{\theta}_s)$	0.335*	0.336*	0.3365	0.3368	0.3370	0.3375	0.3378	0.3383	0.3385
$\frac{M_0}{m\lambda^2} \theta$	0.41*	0.406*	0.399	0.392	0.384	0.374	0.367	0.358	0.357

*Low accuracy because of slow convergence of series expansions in (27) and (28).

REFERENCES

1. P. S. Symonds and C.-F. A. Leth, "Impact of Finite Beams of Ductile Metal", Jour. of the Mech. and Phy. of Solids, 2, 92 (1954).
2. E. H. Lee and P. S. Symonds, "Large Plastic Deformation of Beams under Transverse Impact", Jour. of Appl. Mechs., 19, 308 (1952).
3. P. S. Symonds, "Large Plastic Deformations of Beams under Blast Loading", Technical Report No. 99 of Brown University to Office of Naval Research under Contract N7onr-35801.
4. B. A. Cotter and P. S. Symonds, "The Plastic Deformation of a Beam under a Symmetric Impulsive Loading", Technical Report No. 104 of Brown University to the Office of Naval Research under Contract N7onr-35801.
5. G. I. Taylor, "The Testing of Materials at High Rates of Loading", James Forrest Lecture, Jour. Inst. Civil Engrs., 26, 486 (1946).
6. M. R. Horne, "The Plastic Bending of Mild Steel Beams with Particular Reference to the Effect of Shear Forces", Proc. Roy. Soc., A, 207, 216 (1951).
7. C.-F. A. Leth, "The Effect of Shear Stresses on the Carrying Capacity of I-Beams", Tech. Report No. 107 of Brown University to the Office of Naval Research under Contract N7onr-35801.



Mass/Unit Length = m

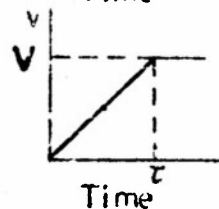
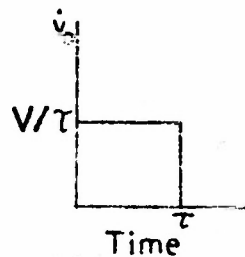


FIG. 1
ACCELERATION AND
VELOCITY OF MID-POINT
OF BEAM

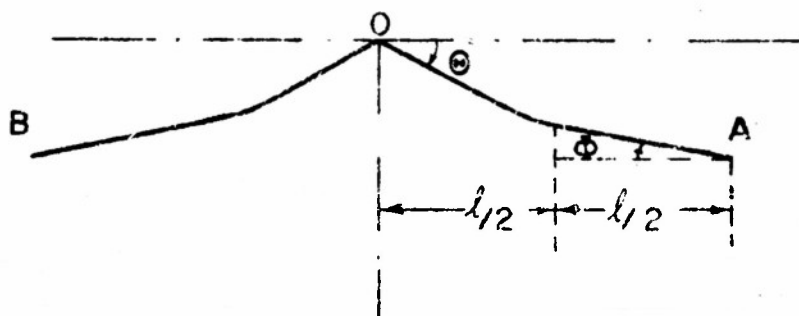


FIG. 2
FINAL DEFORMATIONS
(θ AND ϕ ARE SMALL)

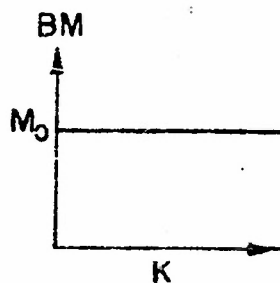


FIG. 3
BENDING MOMENT-CURVATURE
RELATION FOR THE BEAM

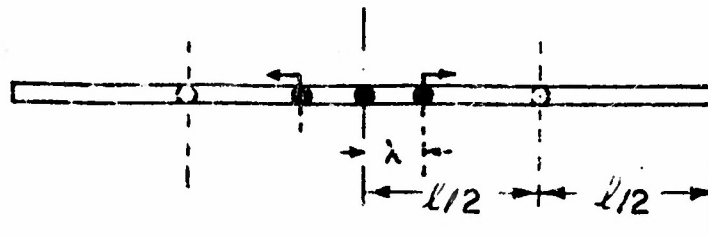


FIG. 4

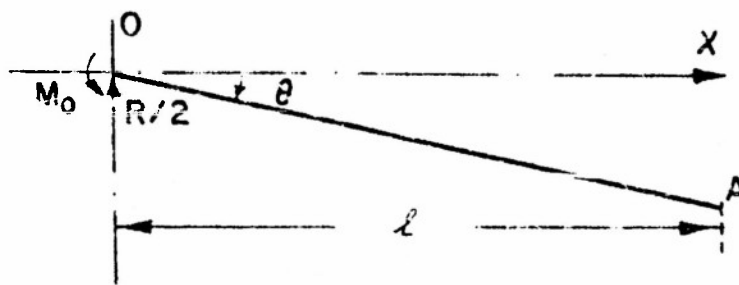


FIG. 5

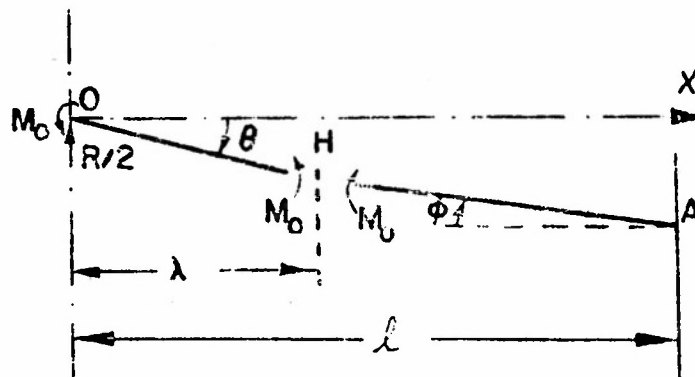
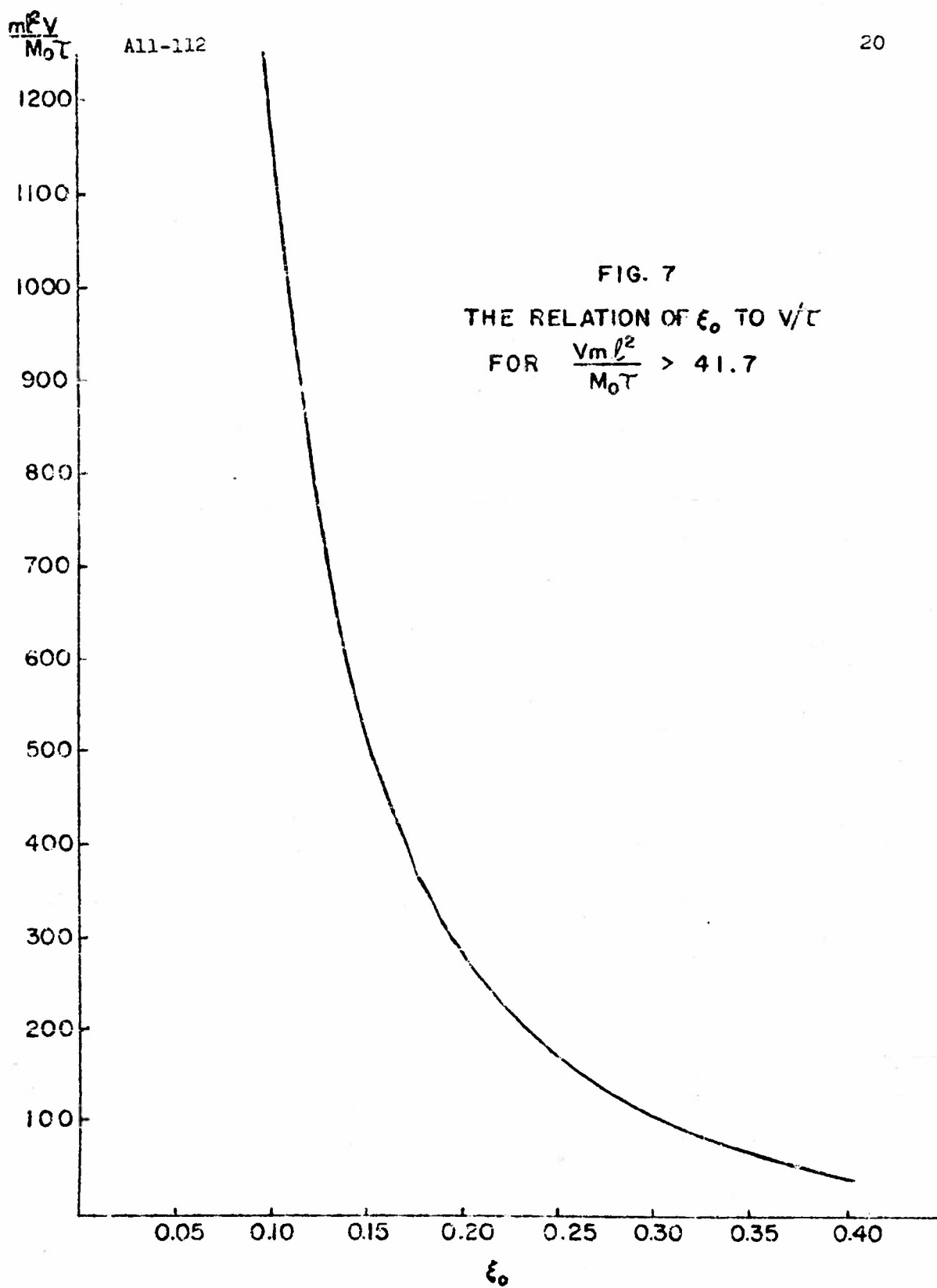


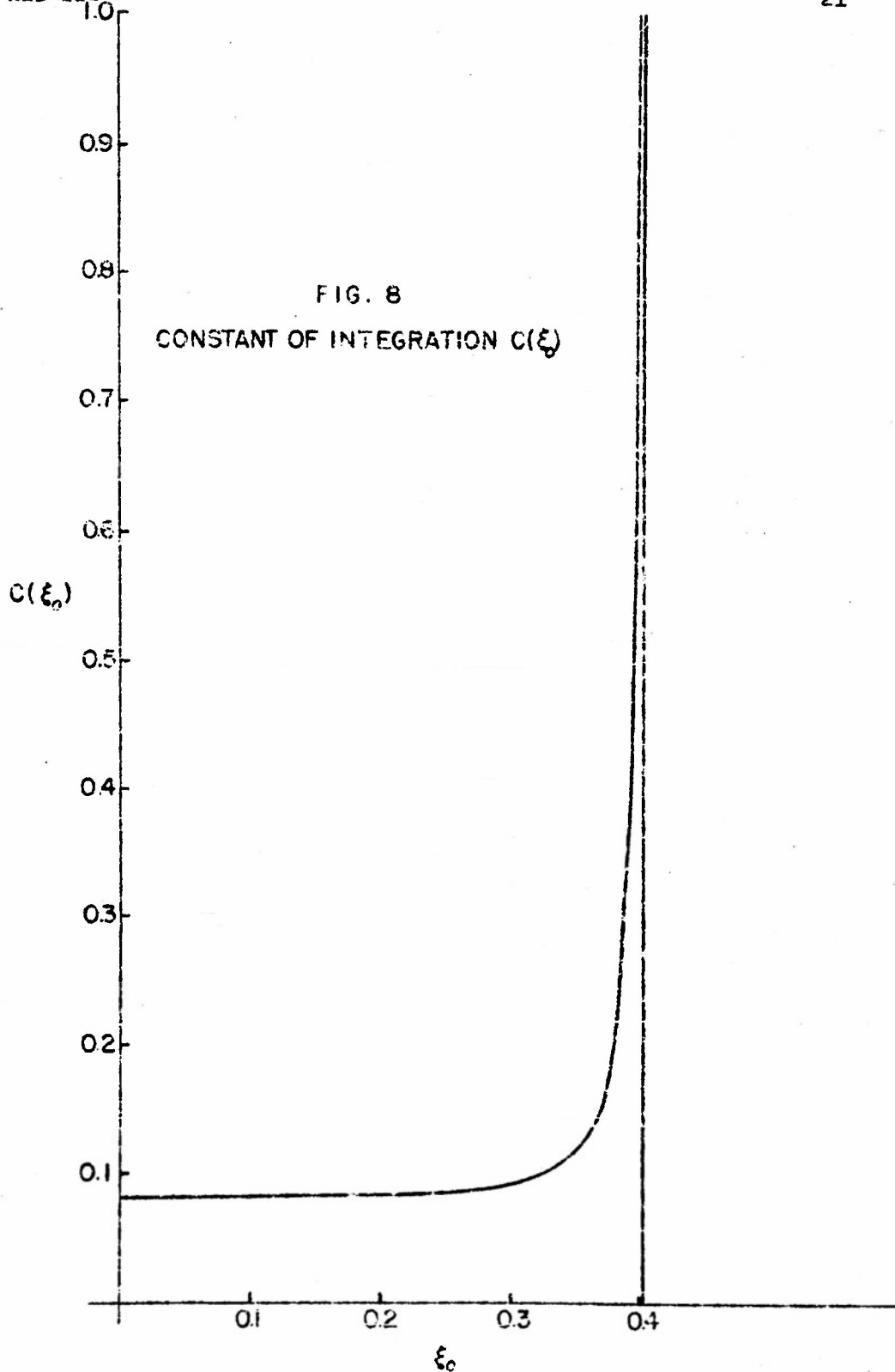
FIG. 6

(R = FORCE IMPRESSED AT O)

A11-112

20





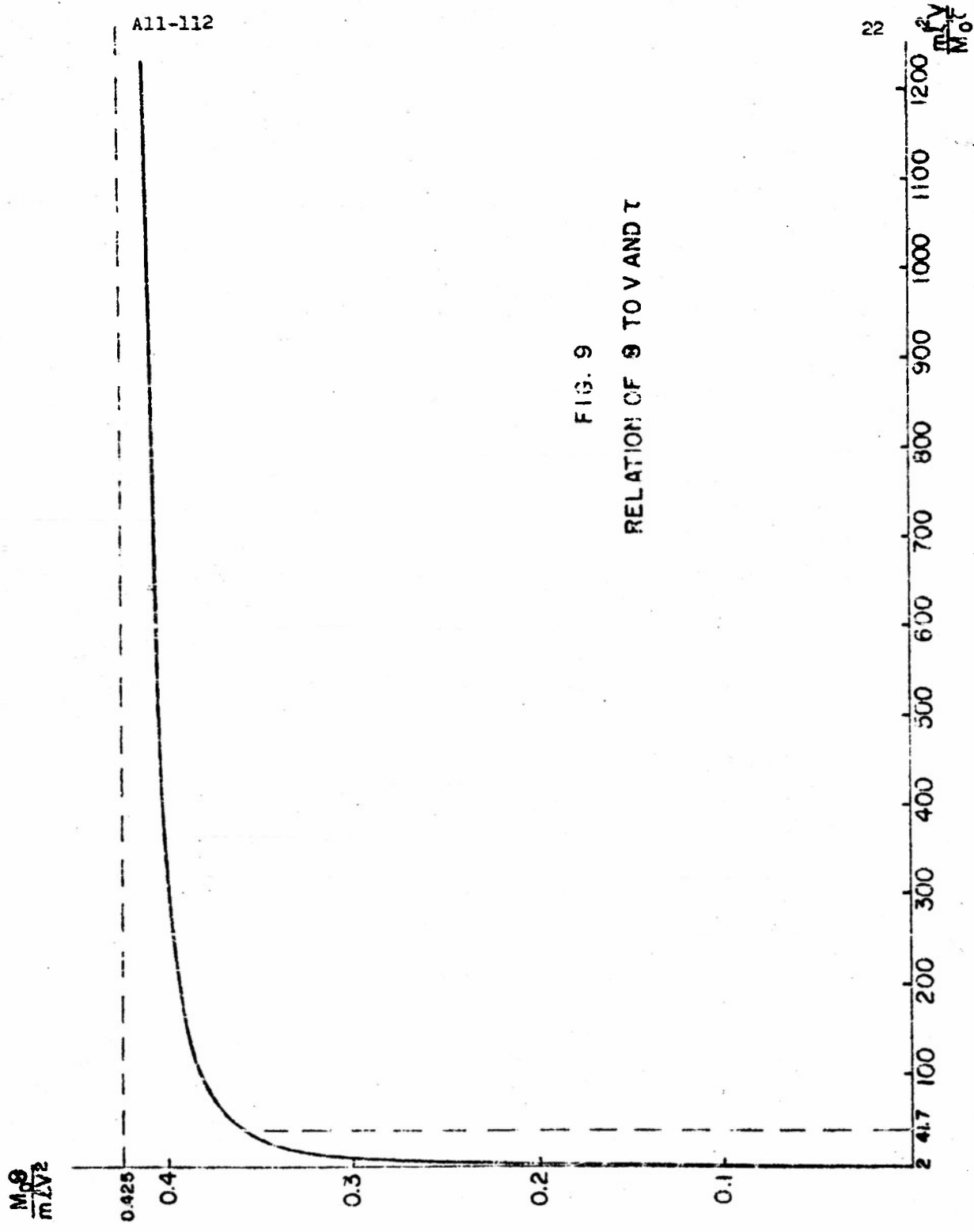


FIG. 9
 RELATION OF \bar{q} TO V AND τ

Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD

37785

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THEREIN.

Reproduced by
DOCUMENT SERVICE CENTER
KNOTT BUILDING, DAYTON, 2, OHIO

UNCLASSIFIED